Supporting an Investigative Approach to Teaching Secondary School Mathematics: A Professional Development Model

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This paper describes a project that supported a group of secondary mathematics teachers in implementing the new Queensland *Mathematics Years 1-10 Syllabus*. The purpose of this paper is to evaluate the effectiveness of the professional development model that was used to assist teachers move towards an investigative approach to "working mathematically". The model integrates a zone-theoretical approach to understanding teacher learning into a framework for designing professional development of mathematics teachers. The effectiveness of the model is evaluated via case studies of teachers' professional learning throughout the project and examination of the impacts on their teaching and assessment practices.

Background

Like all other key learning area syllabuses in Queensland, the recently published *Mathematics Years 1-10 Syllabus* (Queensland Studies Authority, 2004) has an outcomes focus that gives it a different structure from syllabuses previously developed in this state. Instead of specifying what should be learned in particular years or grades of school, the mathematics syllabus is organised around (a) overall learning outcomes that collectively describe attributes of lifelong learners, (b) key learning area outcomes that describe how students think, reason, and work mathematically, and (c) core learning outcomes that describe what students should know and do with what they know in the strands of Number, Patterns and Algebra, Measurement, Chance and Data, and Space. The challenge for teachers implementing the new syllabus lies not only in using the new structure for curriculum planning, but also in designing learning experiences and assessment tasks that take an *investigative approach* to "working mathematically".

An investigative approach to the teaching and learning of mathematics aligns with curriculum reform movements in mathematics education (e.g., National Council of Teachers of Mathematics (NCTM), 2000; Australian Education Council, 1991). Contrasting a traditional rule-based, skill mastery approach to teaching of mathematics, reformist goals include promoting students' communication skills and problem solving capacities, and enabling students to experience the actual processes through which mathematics develops (e.g., conjecture, generalisation, proof, refutation) (Australian Education Council, 1991). These goals resonate with the key learning area outcomes of the Queensland *Mathematics Years 1-10 Syllabus*, which emphasise reasoning, problem solving, communication, and investigation. The importance of an investigative approach to teaching of mathematics has been highlighted in recent classroom based research. For example, the TIMSS Video Study (Hollingsworth, Lokan, & McCrea, 2003) revealed that in Australian classrooms there was little emphasis on developing deep understanding of mathematical concepts or the connections between them. Stacey (2003) described this cluster of features as constituting a syndrome of shallow teaching, where students experience a diet of excessive repetition and problems of low complexity, with very few opportunities for mathematical reasoning. Similar findings were reported by the Queensland School Reform Longitudinal Study (Lingard et al., 2001), a large scale research project involving observations of nearly 1000 lessons across all secondary year levels and subject areas. Mathematics lessons were often found to offer low levels of intellectual quality and connectedness, suggesting that students were given few opportunities to develop higher order thinking and deep understanding, and to appreciate connections between mathematics and the real world. The QSRLS also found that teachers often set assessment tasks that were low in intellectual demand and unconnected to the world outside school.

Implementation of the *Queensland Mathematics Years 1-10 Syllabus* asks teachers to expand their pedagogical and assessment repertoires to include more investigative approaches to “working mathematically”; yet research has revealed how difficult it is for teachers to change their practices to enact curriculum reform (Remillard & Bryans, 2004). This paper reports on a research and development project that supported secondary school teachers in planning and implementing mathematical investigations, consistent with the intent of the new Queensland syllabus. The purpose of this paper is to evaluate the effectiveness of the professional development model that was used to support an investigative approach to mathematics teaching and assessment.

Designing the Professional Development Model

Previous research by Goos (2005a, 2005b) investigated how teachers learn from experience in complex environments, using a theoretical model that re-interprets and extends Vygotsky’s concept of the Zone of Proximal Development (ZPD) to incorporate the social setting (Zone of Free Movement, ZFM) and the goals and actions of participants (Zone of Promoted Action, ZPA). In this model, the ZPD represents *teacher knowledge and beliefs*, and includes teachers’ disciplinary knowledge, pedagogical content knowledge, and beliefs about their discipline and how it is best taught and learned. The ZFM represents constraints within the *professional context*. These may include teacher perceptions of student background, ability and motivation, curriculum and assessment requirements, access to resources, organisational structures and cultures, and parental and community attitudes to curriculum and pedagogical change. The ZPA represents the *sources of assistance* available to teachers that define which teaching actions are specifically promoted. This assistance is typically provided by colleagues and mentors in a school or by formal professional development activities. To understand teacher learning, it is necessary to investigate relationships between these three zones of influence.

Much is known about designing effective professional development to bring about changes in the way that mathematics is taught in schools (Mewborn, 2003). Change is a long term, evolutionary process that can be supported by giving teachers opportunities to engage with mathematical concepts and focus on their own students’ thinking as they struggle to understand these concepts. Professional development is most effective when it occurs in school-based contexts so teachers can try out and validate ideas in their own classrooms. Teachers also need time and opportunities to discuss pedagogical and curricular issues with supportive colleagues as they attempt to implement new practices.

Loucks-Horsley, Love, Stiles, Mundry, and Hewson (2003) created a framework for designing professional development that incorporates the research findings outlined above, and captures the decision making processes that are ideally involved in planning and implementing programs (shaded boxes and "bubbles" in Figure 1).

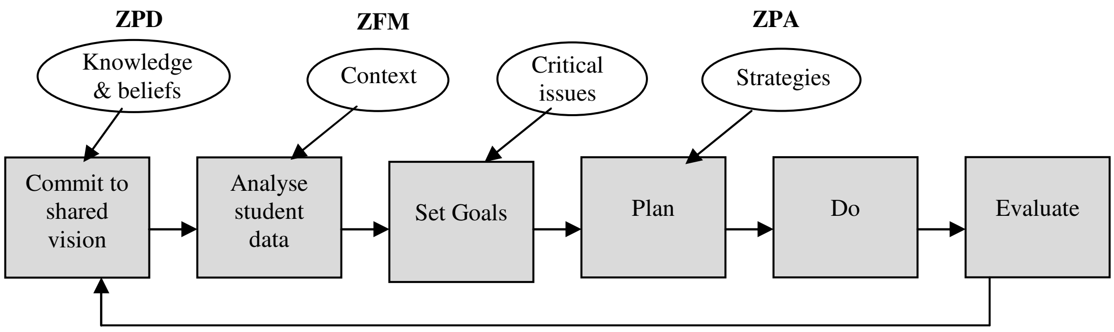

Figure 1. Design framework for professional development.

The planning sequence begins with teachers making a commitment to enhance teaching and learning, thus acknowledging that a tension exists between the current reality and the vision of mathematics teaching offered by new curriculum documents. In practice, it is not always feasible to delay the start of the professional development program until a whole school or group of teachers has established a shared commitment; instead the process of developing this commitment and vision can continue throughout the program and is iterative with other phases of the design. Teacher knowledge and beliefs are an important input into this phase (cf the ZPD discussed above). Analysis of student learning data sharpens the focus on setting targets for improvement and establishing goals for teacher learning and development. It is important here to study the context in order to know who the students are and what teachers know and believe, to identify significant features of the learning environment, and to understand the school's organisational structures and cultures, the local curriculum context, and the views of parents and the community members (cf the ZFM discussed above). The framework suggests anticipating critical issues at the goal setting phase because each of these issues can influence the effectiveness of the program at some point. Planning for professional development can then draw on a wide range of strategies to achieve desired goals (cf the ZPA discussed above).

For this project, we took the Loucks-Horsley et al. (2003) design framework for professional development and considered other literature on effective professional development to plan an overall strategy that is best described as action research. The five key elements of action research (as identified by Loucks-Horsley et al., 2003) guided implementation of the professional development model. The key elements emphasise the need for teachers to own the research project in order for real change to be actualised. They include: (1) teachers devising their own research questions; (2) teachers engaging in the action research cycle; (3) teachers linking with external support mechanisms; (4) teachers working collaboratively; and (5) teachers sharing and disseminating their project with peers. Table 1 summarises how our approach in this project attended to these five key elements.

We envisioned the project as a series of iterative cycles, with teachers coming together to discuss their school-based plans and meet other project teachers, then return to their schools to implement investigative units of work in mathematics, then meet together with project teachers to share their experiences and plan further units of work, as well as an on-site visit to all teachers' classrooms to gain insight into their classroom and school context.

Table 1
Relationship Between Action Research Elements and Project Design

Elements of action research	Project design
Teachers contribute to or formulate their own questions, and collect the data to answer these questions.	Teachers would be encouraged to identify goals relevant to their learning needs and professional context.
Teachers use an action research cycle (set goals, plan, implement, evaluate).	The action research cycle integrates our zone-theoretical model of teacher learning with the Loucks-Horsley et al. (2003) framework for designing professional development.
Teachers are linked with sources of knowledge and stimulation from outside their schools.	The research team would act as a resource for teachers, providing literature on mathematics teaching and assessment as well as exemplary tasks, and advice on collection and analysis of student data.
Teachers work collaboratively.	A pair of teachers would be invited to volunteer from each school so each participant would have continuous collegial support. Pairs would be brought together for professional development meetings with the researchers.
Learning from research is documented and shared.	Teachers would present their work at conferences organised by Education Queensland and attended by key personnel involved in supporting syllabus implementation.

Sources of data to analyse the effectiveness of the project would need to include information about the contexts of the teachers. An audio-taped whole-group interview, where teachers described their teaching situation as well as their personal mathematics teaching history, was planned for the first meeting with the teachers. They would also be asked to complete a Mathematical Beliefs Questionnaire (Frid, 2000) comprising 40 Likert style items about the nature of mathematics, mathematics teaching and mathematics learning. Other planned data sources included the teachers' units of work and student work samples, as well video-taped footage of their classrooms.

Implementing the Professional Development Model

Schools in the region in which the study took place were invited to participate in the project. Schools were specifically requested to nominate pairs of teachers so that they could support each other throughout the project. It was also requested that teachers voluntarily come to this project. The four pairs of teachers who volunteered to participate in the project were from four schools in or near a regional Queensland city. Two schools were in this regional city (Cunningham and Churchill State High Schools), one was in a small rural town nearby (Sugartown State High School), whereas the fourth school was located in a coastal resort approximately 125 km from the regional centre (Seaside State High School). We made three visits to this city to work with the whole group of teachers, in the period from October 2005 to February 2006, each time for two consecutive days. The venue for these meetings was a well equipped computer laboratory in one of the participating schools.

On our first visit to work with the teachers (October 2005) we gathered information about their *knowledge and beliefs* and their *professional contexts* via the Mathematical

Beliefs Questionnaire and the structured audio-recorded whole group interview. This enabled us to map their respective Zones of Proximal Development and Free Movement. The first meeting was also aimed at helping the teachers to identify their personal goals for the project, to demonstrate ways to collect data on their students' beliefs and attitudes towards mathematics, and to provide time and support for them to begin planning units of work to implement in their classrooms. We also wanted to engage these teachers as learners in mathematical investigations on these meeting days. As a *source of assistance* that was deliberately promoting new teaching approaches (Zone of Promoted Action), the professional development model recognized the importance of providing teachers with authentic, practice-based learning opportunities that included examples of mathematical investigations, opportunities to experience these investigations as learners themselves before planning their own investigations and trying them out with their students, and opportunities to share their ideas and experiences with colleagues, including the challenges encountered and their insights into the process. On each of the meeting days, the teachers were provided with some ideas for investigative approaches in particular topics in mathematics. Teachers were also encouraged to share their own ideas of investigative units that they used with their own classes.

In the second visit (November 2005), we facilitated a debriefing discussion of successes and problems each teacher had experienced in implementing their new investigative units. We also modelled the development of assessment criteria for mathematical investigations, and assisted teachers with planning units of work for the start of the 2006 school year. By the third visit (February 2006), sufficient familiarity and trust had been established for us to visit the pairs of teachers in their schools to observe and discuss implementation of the investigative units. The first author visited Sugartown State High School, the second author Churchill and Cunningham, and the third author Seaside. We observed and videotaped at least one lesson during each school visit, and the researcher and teachers then discussed the lesson while watching the video together. This discussion was audio-recorded for later review and analysis. On the second day of this visit we conducted a whole group discussion to evaluate the project and identify implications for extending similar professional development opportunities to teachers in other schools.

Data collected during the project, which comprised interview records, completed questionnaires, videotapes and field notes of lessons, student work samples, and teacher planning documents, were analysed by interpreting the particular circumstances under which elements of the professional development model (Figure 1) were "filled in" with specific people, actions, places, and meanings.

Effectiveness of the Professional Development Model

We evaluated the effectiveness of the professional development model by examining how the teachers negotiated opportunities and hindrances in pursuing investigative approaches to mathematics teaching and assessment. Relevant information is summarised in Figure 2.

From Figure 2, it can be seen that the four pairs of teachers came from quite contrasting school contexts, with three pairs of teachers being rated as having student-centred beliefs and one pair having teacher-centred beliefs. Three of the schools had quite structured and traditional approaches to teaching of mathematics, with one school having a very flexible approach.

Element of PD Model	Schools and Teachers			
	Sugartown Skye & Chris	Seaside Val & Shanti	Cunningham Peter & Ron	Churchill Tony & Ralph
Knowledge & beliefs (ZPD)	Qualified in maths & maths ed. Student-centred beliefs	Mixed quals in maths & maths ed. Student-centred beliefs	Qualified in maths & maths ed. Student-centred beliefs	Mixed quals in maths & maths ed. Teacher-centred beliefs
Professional context (ZFM)	Low achieving Ss Poorly resourced Classes streamed HOD supportive of change Low SES Little parental support for school	Test & textbook dominated practices Poorly resourced Classes streamed via frequent tests Organisational culture resistant to reform approaches	High achieving Ss take Project Maths extension Other subjects use traditional methods Well resourced Flexible timetable Ts plan & teach together	School has strong academic reputation Lecture approach + streamed classes Other Ts resistant to change Tony as HOD of Middle Years seeks curriculum reform
Goals	Engaging learners in meaningful mathematics	Making assessment more authentic and practical	Making Project Mathematics mainstream	Integrating maths with other KLAs

Figure 2. Teacher characteristics, contexts, goals, and longer term impact.

Beyond the information presented in Figure 2, the following summary describes the context and beliefs of the four pairs of teachers when they first started in the project.

Skye and Chris: student-centred beliefs, teaching in a school that has little parental support and low achieving students. However, at this school, a new Year 8 class of students who demonstrated low mathematics outcomes had been created, and the project felt supported at their school to try new approaches with this class.

Val and Shanti: student-centred beliefs, teaching at a school where traditional approaches to mathematics teaching were expected. These two teachers felt that there was a better way to teach mathematics than what was expected at their school. They felt comfortable trying new ways of teaching in their own classrooms, but felt their fellow mathematics teachers disapproved of such approaches.

Peter and Ron: student-centred beliefs, were already implementing a mathematics program which took an investigative approach with extension classes. The goal for these two teachers was to try to make their investigative maths classes integrate into the mainstream classes.

Tony and Ralph: teacher-centred beliefs, high academic student outcomes. Although these two teachers came from the same school, they had quite different reasons for volunteering for the project. One was a science teacher who had just been made middle years coordinator. This teacher wanted to support fellow teachers in the middle years to take a more integrated approach to teaching. The second teacher was teaching a low achieving Year 9 mathematics class and he was hoping to develop new approaches for engaging these learners.

Information in Figure 2 highlights the diversity of professional contexts featuring potentially helpful and unhelpful influences on the teachers participating in the project, however, it does not show how these influences interacted to either support or hinder teacher learning. Therefore, an example of such interactions is provided in the abbreviated case study that follows.

Case Study of Teacher Learning: Skye and Chris

Skye wanted to take a more investigative approach to teaching her new Year 8 Practical Mathematics class of students who were not achieving success in regular mathematics classrooms. Chris was mainly teaching senior classes and wanted help in planning new programs and devising new forms of assessment. Questionnaire responses revealed that they both held similar beliefs about the nature of mathematics, and mathematics teaching and learning. For example, they agreed that there are many ways of interpreting and solving a problem, and that it is important to encourage students to build their own mathematical ideas. However, other responses showed they were uncertain about the benefits of more traditional approaches such as memorisation and practice. This suggests that Skye and Chris were interested in moving towards more student-centred, investigative teaching practices, but that they needed to try out these practices with their own classes to find out whether this would lead to improved learning.

Skye and Chris stated that the most frustrating obstacle in their professional context was the students themselves, and their apparent lack of interest in learning. This was evident in the students' disruptive and uncooperative behaviour, and their frequently stated belief that they were "dumb" and simply could not do mathematics. The experience of teaching unmotivated students led these teachers to formulate a goal of *engaging learners*, or, as Skye explained, "for them to learn maths without being terrified of it". Both saw investigations as a way of presenting mathematics differently that would allow them to make mathematics more interesting for students by engaging them in purposeful tasks with real world relevance.

With full support of their Head of Department, Skye and Chris decided to team-teach the Practical Mathematics class and their teaching timetable was altered to enable this to occur. Skye and Chris's first unit of work asked students to investigate whether it is more economical to buy groceries in Sugartown or drive to the larger regional centre nearby. After reflecting on the mixed outcomes of their first unit, they then planned a unit that they hoped would more closely connected to students' lives. Their "School Rage" investigation asked students to create a Top 20 song list for the school radio station, based on a survey of students attending the school. To make the task more realistic, a letter from the "radio station manager" (one of the mathematics teachers) was given to the students asking for assistance in designing a new radio program similar to the Rage Top 20. The group submitting the best quality report would have their Top 20 songs played on the radio station during a designated lunchtime. Thus the task had an authentic purpose and a real audience comprising the entire school community. Core learning outcomes embedded in this task related to designing and carrying out data collections, using data record templates, organising data and creating suitable displays, making comparisons about data, and working with whole numbers, fractions and percentages.

Classroom observations confirmed the teachers' judgment that students were deeply engaged in the investigation. Overheard comments suggested that the students welcomed this new approach. In their own evaluation of the units, Skye and Chris not only identified the benefits for the students (engagement, confidence, alternative opportunities to demonstrate their learning) but also the challenges the new approach presented to them. They were now spending more class time responding to unanticipated ways students tackled investigations, often by asking questions to scaffold students' thinking, such as "What does it mean if you include the same person twice in your survey?" and "What if this person votes for two different songs?" Skye pointed out that she welcomed such unexpected responses as she regarded it as a sign of growth of sophistication in students' thinking.

Skye and Chris identified several reasons why they had been successful in implementing an investigative approach. They often emphasised the importance of taking into account the students' prior experience and interests, and the local context of the school and community. Access to sample investigations was critical, as was access to human resources in the form of a supportive school administration team, a network of like minded mathematics teachers across the schools participating in the project, and their teaching partner. Planning and teaching as a team, rather than individuals, was a significant benefit for both teachers because they recognised that this reduced their workload, expanded their repertoire of teaching strategies, and provided opportunities for mutual observation and feedback. Skye and Chris's professional learning experience is summarised by the relationships between their knowledge and beliefs (ZPD), professional context (ZFM), and sources of assistance (ZPA), shown in Figure 3. Although they experienced hindrances within their professional context, productive tensions between aspects of the context and their pedagogical beliefs led them to formulate and pursue the goal of engaging learners.

Discussion

Our evaluation of the professional development model was guided by the zone-theoretical model of teacher learning outlined earlier in the paper. For each of the four case studies of pairs of teachers, we were able to identify a different configuration of *teacher knowledge and beliefs* (Zone of Proximal Development), *professional contexts* (Zone of Free Movement), and *sources of assistance* (Zone of Promoted Action), and how these factors came together to shape opportunities for teacher learning. A sample case study illustrated one such configuration. Although there were some differences in the teachers' espoused beliefs about mathematics and how it is best learned and taught, all of them came to the project looking for inspiration and ideas about taking a more investigative approach to their classroom practice, and some were already experimenting with investigative approaches to mathematics teaching. Nevertheless the teachers commented that it was unlikely significant change would have occurred without the impetus provided by this project, because the opportunity to participate validated the changes in teaching and assessment practices that they wanted to achieve. The credibility and authority they gained from participation were vital for helping them deal with relatively inflexible organisational structures and resistance from more traditionally minded mathematics teachers in their schools. Several of the teachers also commented that working with university researchers had enhanced their status as professionals in the eyes of their colleagues. Although these teachers worked in diverse professional contexts that offered both opportunities for, and hindrances to, innovation, all were able to draw on their knowledge and beliefs and the sources of assistance available to them to plan and implement teaching approaches consistent with the intent of the new syllabus.

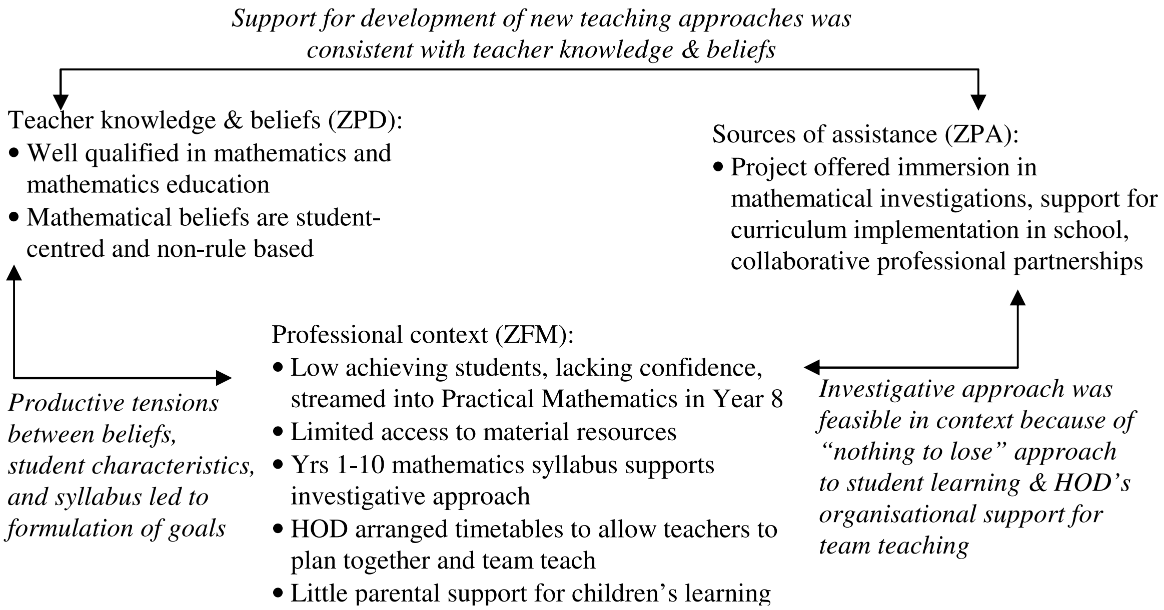

Figure 3. Relationships between professional learning factors for Skye and Chris.

With regard to the professional development model, three clusters of features seemed to contribute to the overall outcomes of the project. The first cluster centres on *professional development processes* involving formulation of realistic goals, provision of long term experiences rather than one-off workshops, and opportunities for teachers to teach and assess student learning during the units implemented. A second cluster of features acknowledges the *resources* required, such as curriculum materials that align with the syllabus, time for planning and reflection with colleagues, and administrative support and commitment. The third set of features focuses on *roles and relationships*, such as the voluntary nature of teachers' participation, acknowledgement of the equal but different contributions made by teachers and researchers, and the importance of broadening participants' perspectives beyond the scope of classroom or school.

Conclusion

Loucks-Horsley et al. (2003) identify several critical issues that must be taken into account when planning a professional development program, preferably in the goal setting phase (see Figure 1). Although we were conscious of these issues throughout the project, their influence is best analysed by looking to the future and asking how might we improve on the conduct of this project in the light of our experiences and what are the implications for extending similar professional development opportunities to secondary mathematics teachers in other schools. One critical issue concerns the need for *building a professional culture* characterised by a strong vision of learning and collegial interactions between teachers. A second issue involves *developing leadership* in teachers who have the capacity to improve the quality of teaching and learning in their schools. Often the most powerful leadership exercised by teachers is simply in modelling new practices for colleagues to demonstrate that they actually work with students. *Building capacity for sustainability* is necessary to ensure that any changes achieved within the life of a professional development project are sustained after it ends. Similarly, *scaling up* is a vital concern for education systems as teachers and school districts implement new teaching and learning approaches. Finally, *gaining public support* for mathematics education is necessary for building consensus around curriculum and pedagogical reform, thus leading to a more informed public understanding of effective methods for teaching mathematics and of the role of mathematics in preparing young people for productive work, leisure, and citizenship.

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